Energy efficiency in spacetime of axially symmetric magnetized Reissner-Nordström black hole

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The spacetime metric

The spacetime metric describing axially symmetric magnetized Reissner-Nordström black hole (Gibbons et al)

$$ds^2 = H\left(-Fdt^2 + F^{-1}\,dr^2 + r^2 d\theta^2\right) + H^{-1}\,r^2\sin^2\theta\,(d\phi - \omega dt)^2\,, \quad (1)$$

where $F = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$ $\quad \omega = -\frac{2QB}{r} + \frac{1}{2}QB^3\,r(1 + F\cos^2\theta)$

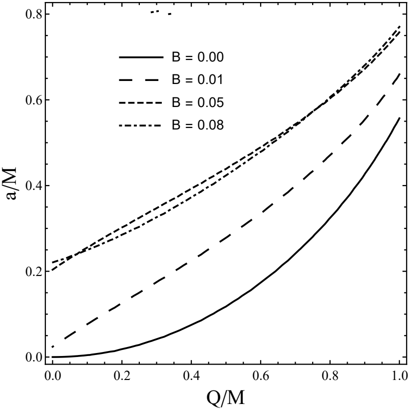
$$H = 1 + \frac{1}{2}B^2(r^2\sin^2\theta + 3Q^2\cos^2\theta) + \frac{1}{16}B^4(r^2\sin^2\theta + Q^2\cos^2\theta)^2$$

Note that M and Q respectively correspond to the black hole mass and charge, while B refers to as the magnetic field.

The black hole horizon $r_\pm = M \pm \sqrt{M^2 - Q^2}$

Typically $a \rightarrow g_{t\phi}$ and Similarly $B \rightarrow g_{t\phi}$

The magnetized RN BH can mimic the rotating Kerr BH



The plot shows the values of spin parameter a as a function of black hole charge Q for which the degeneracy appears for value of the ISCO. For a given value of B the Kerr and magnetized Reissner-Nordström black hole geometries have the same ISCO.

Curvature properties

$$\mathcal{K} = \frac{N(r,M,Q,B,\theta)}{\left[B^4(Q-r)^2(Q+r)^2\cos^4\theta+2\,B^2\left((Q^2r^2-r^4)B^2+12\,Q^2-4\,r^2\right)\cos^2\theta+(B^2r^2+4)^2\right]^6 r^8},$$

where $N(r, M, Q, B, \theta)$ is very long and complicated expression for explicit display. It is obvious that $\mathcal{K}$ contain two sorts of identical singularities, one is real at $r = 0$, similar to the Schwarzschild spacetime, and the other is imaginary, which comes from the polynomial in r in the denominator.

Electromagnetic field of the magnetized RN BH

Vector potential of the electromagnetic field around the black hole has the form as

$$A \;=\; A_t dt + A_\phi (d\phi - \omega dt)\,, \tag{2}$$

where components of the vector potential of the electromagnetic field are given as follows:

$$\boxed{A_t = -\frac{Q}{r} + \frac{3}{4} Q B^2 r \left(1 + F\cos^2\theta\right)}$$

$$\boxed{A_\phi = \frac{2}{B} - H^{-1}\left[\frac{2}{B} + \frac{1}{2} B (r^2 \sin^2\theta + 3Q^2\cos^2\theta)\right]}$$

Electromagnetic field of the magnetized RN BH

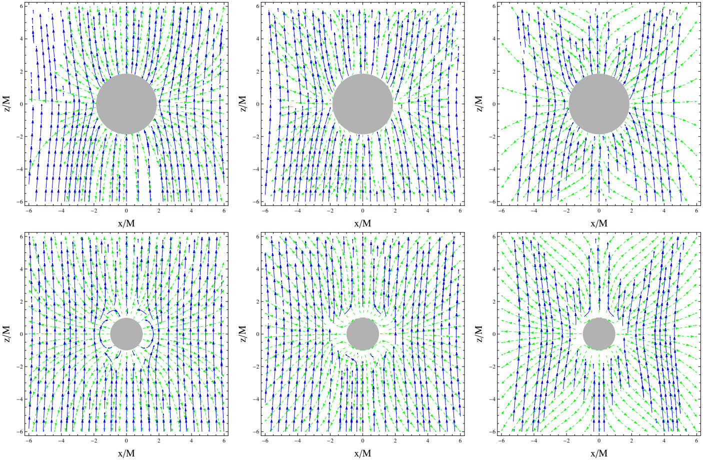
$$E^{\hat{r}} = H^{-1}B\Bigg\{\left(\left(\frac{Q}{Br^2}+\frac{3}{4}QB\left(1+\left(1-\frac{Q^2}{r^2}\right)\cos^2\theta\right)\right)\right)+H^{-1}\omega r\left(H^{-1}\left[1+\frac{B^2}{4}\left(r^2\sin^2\theta+Q^2\cos^2\theta\right)\right]\right.$$
$$\left.\times\left[2+\frac{B^2}{2}\left(r^2\sin^2\theta+3Q^2\cos^2\theta\right)\right]-1\right)\sin^2\theta\Bigg\},$$

$$E^{\hat{\theta}} = \frac{BH^{-1}}{r}\Bigg\{f^{-1/2}\omega H^{-1}\left(H^{-1}\left[r^2-3Q^2+\frac{B^2}{4}\left(r^2-Q^2\right)\left(r^2\sin^2\theta+Q^2\cos^2\theta\right)\right]\right.$$
$$\left.\times\left[2+\frac{B^2}{2}\left(r^2\sin^2\theta+3Q^2\cos^2\theta\right)\right]-r^2+3Q^2\right)-\frac{3}{2}QBr\left(\frac{1}{f}\right)^{-1/2}\Bigg\}\sin\theta\cos\theta\,,$$

$$B^{\hat{r}} = -\frac{B(f/H)^{1/2}}{r^2\left(Hf-H^{-1}\omega^2r^2\sin^2\theta\right)^{1/2}}\left(H^{-1}\left[r^2-3Q^2+\frac{B^2}{4}\left(r^2-Q^2\right)\left(r^2\sin^2\theta+Q^2\cos^2\theta\right)\right]\right.$$
$$\left.\times\left[2+\frac{B^2}{2}\left(r^2\sin^2\theta+3Q^2\cos^2\theta\right)\right]-r^2+3Q^2\right)\cos\theta\,,$$

$$B^{\hat{\theta}} = \frac{Bf\,\sin\theta}{\left(H^2f-\omega^2r^2\sin^2\theta\right)^{1/2}}\left\{H^{-1}\left[1+\frac{B^2}{4}\left(r^2\sin^2\theta+Q^2\cos^2\theta\right)\right]\left[2+\frac{B^2}{2}\left(r^2\sin^2\theta+3Q^2\cos^2\theta\right)\right]-1\right\}.$$

The configuration of electromagnetic field



The configuration of electromagnetic field in the vicinity of a magnetized Reissner-Nordström black hole. Blue and green lines describe the magnetic and electric fields, respectively. Meanwhile, the horizon is shown as a gray-shaded area. For the figure: first, second, and third columns respectively refer to $B = 0.01$, $B = 0.05$, and $B = 0.1$ in the case of fixed $Q = 0.1$ (top panel) and $Q = 0.5$ (bottom panel).

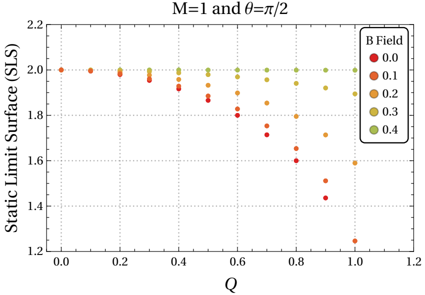
The values of the ISCO radius (r_i), $\mathcal{L}_i$ and $\mathcal{E}_i$ of the neutral test particles orbiting on the ISCO around the MRN BH are tabulated for different values of Q and B.

		B				
Q		0.00	0.01	0.04	0.08	0.10
0.0	r_i	6.00000	5.91997	5.32285	4.75062	4.56610
	$\mathcal{L}_i$	3.46410	3.48144	3.70022	4.28076	4.72620
	$\mathcal{E}_i$	0.94281	0.94505	0.97676	1.07860	1.16614
0.1	r_i	5.98497	5.74518	5.13935	4.68373	4.52912
	$\mathcal{L}_i$	3.45928	3.52444	3.81248	4.35519	4.72863
	$\mathcal{E}_i$	0.94267	0.95548	1.01262	1.13196	1.21873
0.2	r_i	5.93957	5.57969	4.97438	4.60136	4.47579
	$\mathcal{L}_i$	3.44471	3.55248	3.89573	4.40613	4.72071
	$\mathcal{E}_i$	0.94227	0.96522	1.04589	1.18113	1.26735
0.3	r_i	5.86278	5.41358	4.81505	4.50213	4.40397
	$\mathcal{L}_i$	3.42006	3.56508	3.95200	4.43514	4.70207
	$\mathcal{E}_i$	0.94158	0.97416	1.07653	1.22626	1.31222

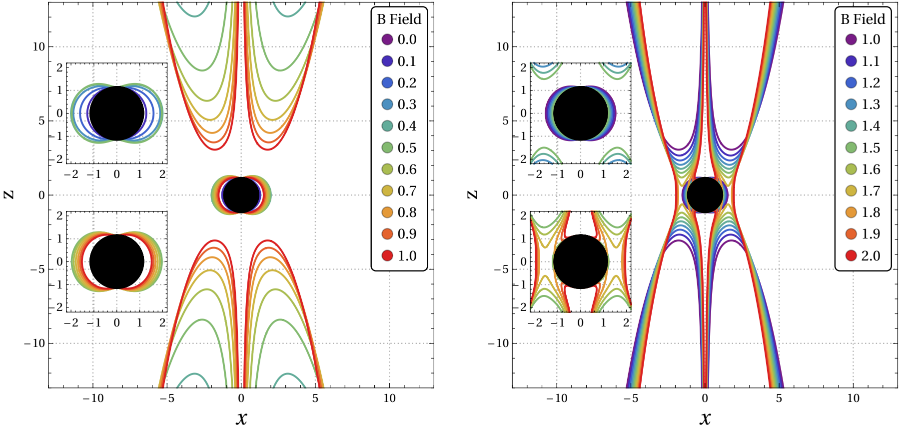
The charged particle dynamics for fixed $B = 0.1$

	q			
Q	0.01 -0.01	0.05 -0.05	0.10 -0.10	0.50 -0.50
0.1	4.52982 4.52838	4.53219 4.52504	4.53413 4.52009	4.49620 4.46278
0.2	4.47615 4.47539	4.47708 4.47335	4.47706 4.46989	4.41627 4.42156
0.3	4.40338 4.40401	4.40295 4.40367	4.40045 4.40222	4.31340 4.36613
0.5	4.19223 4.19496	4.18596 4.19964	4.17649 4.20417	4.02520 4.20550
1.0	2.95865 2.97713	2.92014 3.01258	2.86902 3.05416	2.30516 3.29255

Static limit behaviour for the magnetized RN BH

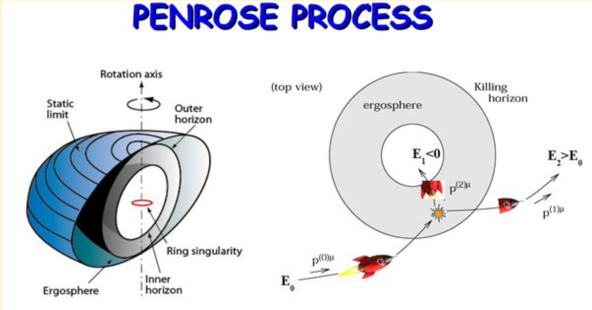


The variation of the outermost static limit surface as a function of black hole charge parameter Q for various combinations of B is shown. B ranges from 0 to 0.4 in the *left panel* and from 0.4 to 1.0 in the *right panel*. Here, the mass parameter $M = 1$ and $\theta = \pi/2$.



For varying combinations of magnetic field B, the ergo region for the extremal axially symmetric magnetised black hole case is shown on the $x - z$ plane. In the *left panel* , the zoomed picture for cases when the magnetic field varies between 0 and 0.5 is shown in the *upper* inset plot. The *lower* inset plot, on the other hand, is a zoomed picture for cases when the magnetic field varies between 0.5 and 1.0. Similarly, in the *right panel*, the zoomed picture for cases when the magnetic field varies between 1.0 and 1.5 is shown in the *upper* inset plot. The *lower* inset plot, on the other hand, is a zoomed picture for cases when the magnetic field varies

Recent modern astronomical observations show that the outflows that can have energies in the range of $E \approx 10^{42} - 10^{47}\ \mathrm{erg/s}$ from AGN in the form of winds and jets have been observed via X-ray, γ-ray and VLBI observations. There is ofcourse the relevance of the charged particle motion with these particle outflows coming out from AGN. Thus, one can explore Penrose process for modeling the central engine in quasars, AGNs and UHECRs.



$$E_{rot} = Mc^2 - \frac{Mc^2}{\sqrt{2}}\left(1 + \sqrt{1 - \frac{a^2}{M^2}}\right)^2 \approx 29\% \text{ of the black hole energy}$$

Magnetic Penrose process (Model)

$$\begin{cases} E_1 = E_2 + E_3 \\ L_1 = L_2 + L_3 \end{cases} ; \begin{cases} m_1 = m_2 + m_3 \\ q_1 = q_2 + q_3 \end{cases} \quad \text{and} \quad \boxed{m_1 u_1^\mu = m_2 u_2^\mu + m_3 u_3^\mu}.$$

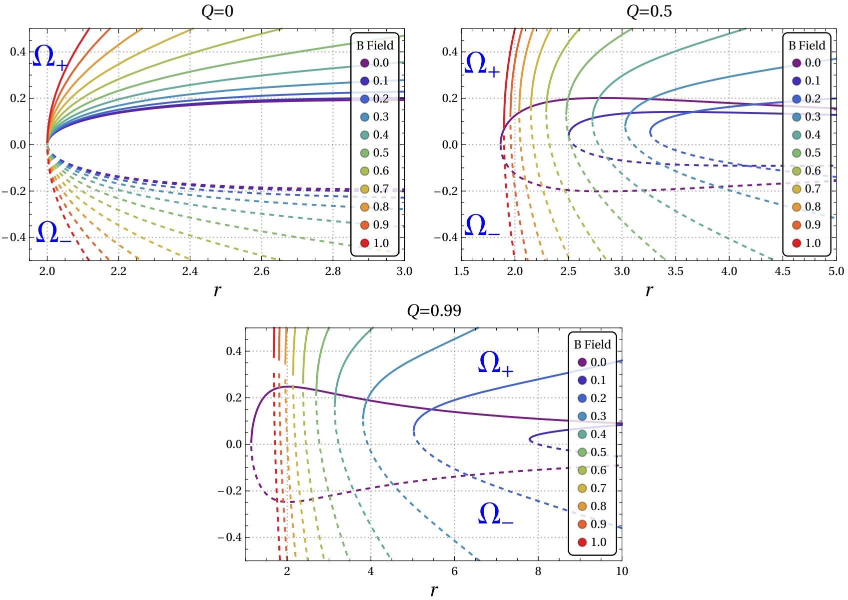
where the condition, $E_2 < 0$ and $E_3 \gg E_1$ is always satisfied as per Penrose mechanism.

For $q_2 + q_3 = 0$ the energy efficiency of MPP takes the form as

$$\boxed{\eta = \left(\frac{\Omega - \Omega_-}{\Omega_+ - \Omega_-}\right)\left(\frac{g_{tt} + \Omega_+ g_{t\phi}}{g_{tt} + \Omega\, g_{t\phi}}\right) - 1 - \frac{q_3 A_t}{E_1}\,.}$$

Note that the condition that the vector $\mathbf{u} \sim \xi_{(t)} + \Omega \xi_{(\phi)}$ is timelike requires $\Omega_- < \Omega < \Omega_+$.

Magnetic Penrose process



Energy efficiency for magnetic Penrose process

For $q = 0$ and $q \neq 0$, η respectively reads as follows:

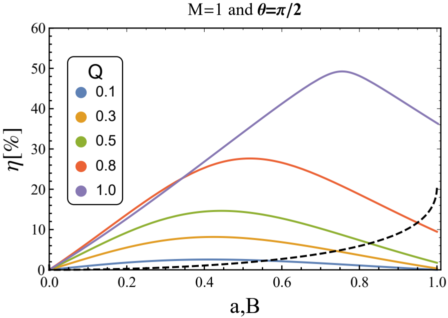
$$
\begin{aligned}
\eta|_{q=0} &= \frac{1}{2\left(4 + B^2\left(1+\sqrt{1-Q^2}\right)^2\right)}\left[\left(-8BQ\left(1+\sqrt{1-Q^2}\right)\right.\right. \\
&\times \left(4 - B^2\left(1+\sqrt{1-Q^2}\right)^2\right) \\
&+ \left.\left(4 + B^2\left(1+\sqrt{1-Q^2}\right)^2\right)^2\right)^{1/2} \\
&- \left.\left(4 + B^2\left(1+\sqrt{1-Q^2}\right)^2\right)\right], \qquad (3)
\end{aligned}
$$

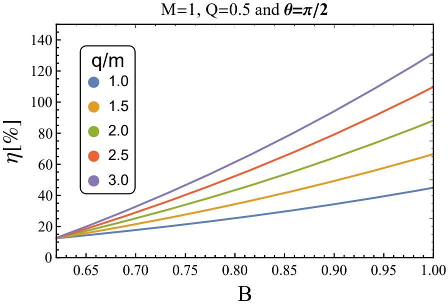
and

$$
\eta|_{q\neq 0} = \eta|_{q=0} - \frac{q}{E_1}\left[-\frac{Q}{1+\sqrt{1-Q^2}} + \frac{3}{4}QB^2\left(1+\sqrt{1-Q^2}\right)\right]. \qquad (4)
$$

For further analysis we shall for simplicity consider $E_1/m_1 = 1$.

Energy efficiency for Magnetic Penrose process





Plot shows the energy efficiency as the function of B in the equatorial plane, $\theta = \pi/2$. Left panel: efficiency $\eta[\%]$ is plotted for various combinations of Q in the case with neutral particle, i.e. $q = 0$. Right panel: efficiency $\eta[\%]$ is plotted for various combinations of q/m in the case of fixed $Q = 0.5$. Note that dashed line in the left panel shows the energy efficiency for rotating Kerr black hole.

Conclusion

- the combined effect of black hole electric charge and magnetic field can mimic black hole spin up to $a/M \approx 0.9$.
- the maximum value of the efficiency reaches up to the value greater than 50 % which is comparable value for Kerr black hole case (i.e. around 20%, which is purely due to the black hole's spin). This is a remarkable property of the axially symmetric magnetized RN BH.
- energy efficiency can exceed 100 % in the case of $q \neq 0$. This happens because the second term of Eq. (4) gives rise to the main contribution so that η reaches its large values.
- the magnetized RN BH solution could be one of the candidates for all its astrophysical applications for modeling the central engine in quasars, AGNs and UHECRs.

Thank you!